PREFACE TO THE FIRST EDITION

This book has evolved from a set of notes that were originally prepared as a replacement of lectures for off-campus students who could not attend class. Consequently the material is written for the student who is studying without the aid of an instructor. In the early stages of development the notes were used in conjunction with lectures on campus. During the later stages the campus lectures were discontinued when it became apparent that the notes had progressed to where they could be used successfully without assistance from the instructor. The notes were also used at Stanford University and at the University of Arizona to enhance the clarity of the final version of the book.

The book has been designed to meet the needs of students taking a course in conduction heat transfer at the first-year graduate level or as a senior elective. (The student is expected to have already taken a first course in heat transfer.) One purpose in taking a course in conduction is to gain the ability to solve more advanced heat-transfer problems. Gaining this ability will require an increased facility in engineering problem formulation and a higher level of mathematics than is usually achieved at the undergraduate level.

The engineering involved in solving conduction problems is contained in the mathematical modeling and in the interpretation of the final mathematical results. This type of material is most effectively taught by the instructor in the classroom and most effectively learned by working comprehensive problems. The instructor can select specific applications which he feels will be of particular interest to his students and then build mathematical models in class where he must defend his assumptions in the face of student questions. Challenging problems can be assigned to be worked on outside of class. Discussion of the assumptions and results is an excellent way to develop an engineering appreciation of conduction.

The optimum situation in a heat-transfer course is for the entire class to have the necessary mathematical background to solve the advanced problems that are being discussed. The instructor can then devote his efforts to the engineering aspects of problem solving. It has been the author's experience, however, that the instructor is often faced with a class that does not yet have enough mathematical tools. Either the student has not yet had the necessary mathematics or, at best, is currently enrolled in a course that may provide some of the information he needs (although several weeks too late perhaps). This usually means that the conduction course must teach mathematics as well as engineering.

One of the intents of this book is to remove the burden of having to discuss mathematical techniques in the heat-transfer classroom, allowing the instructor to concentrate on engineering aspects while leaving the mathematical details to be learned by the student from the textbook outside of class. The book is written for the student who has absolutely no background in the mathematical topics required to solve the conduction heat transfer examples contained in Chaps. 2 through 9. It is designed to give the engineer a palatable exposure to enough additional mathematics to enable him to solve problems that were beyond his undergraduate heat-transfer course. The problems selected for discussion have been chosen because they elucidate mathematical procedures and because they also have significant engineering applications. An engineering approach to mathematics is taken throughout the book. Existence and uniqueness theorems are not presented because, if the model is physically correct, there must be a unique solution. Indeed, the engineer's physical insight into the problem gives him a valuable tool that is not often used by the pure mathematician. The book attempts to bridge the gap between mathematician and engineer.

The book is not intended to be a compilation of existing classical solutions to conduction problems. An extensive supply of such solutions may be found in *Conduction of Heat in Solids,* 2d ed., by Carslaw and Jaeger.[1] An understanding of the material in the first seven chapters of this book is essential for proper and efficient use of Carslaw and Jaeger's treatise.

It is important that the engineer have a balanced view of methods for solving problems. An overemphasis of exact, analytical methods at the expense of approximate, computer oriented techniques would not reflect the trends in technology. Consequently this book does not attempt to cover all possible analytical methods that might be used, nor does it cover the topics in as great a depth as if it were a mathematics textbook. The variety and depth of presentation of the exact methods of solution have been chosen to give the engineer an appreciation of the kinds of problems that can be solved exactly and the amount of work required to do so. A good portion of the text is devoted to approximate methods that are suitable for the digital computer.

It is expected that the student will find the first chapter to be almost entirely a review of topics he was exposed to in his first course in heat transfer. Familiarity with this material is essentially the only prerequisite

[1] Oxford University Press, Fair Lawn, NJ, 1959.

required for the remainder of the book. The amount of time devoted to this chapter will depend upon the student's background.

Bessel functions arise in the mathematical solution of circular fin problems and from the solution of partial differential equations in cylindrical coordinates. Chapter 2 is intended to serve as an introduction to these functions. In so doing, it should also be of value when considering other types of special functions which often appear in heat-transfer analyses.

Separation of variables is the most common technique for solving partial differential equations and is discussed in Chap. 3. Emphasis is given to the types of boundary conditions found in engineering practice. These are often omitted in a mathematics course.

The extension of basic solutions to more complicated problems using the notion of superposition is considered in Chap. 4. This idea is discussed in detail due to its usefulness in convection as well as in conduction and in other engineering systems.

Problems with periodic boundary conditions often occur in engineering. The method of complex combination discussed in Chap. 5 is very useful in obtaining the sustained solution to such problems.

The Laplace transformation is introduced in Chap. 6 primarily because of the ease with which it can handle the semi-infinite solid. This technique can also be used as an alternative to separation of variables.

Although the use of nondimensional quantities is widespread in heat transfer and other engineering fields, it is rarely considered as an analytical tool in its own right. Chapter 7 discusses normalization as a mathematical tool for simplifying problems and for obtaining deeper insights without going through the complete mathematical solution of a problem. This chapter should be read in parallel with the earlier chapters of the book.

The first seven chapters should provide the student with a good idea of the types of problems that can be solved exactly and some of the possible methods of obtaining solutions. There are many problems which arise in engineering practice, however, that can not be handled exactly. Therefore the remainder of the book is devoted to the discussion of numerical methods that can be carried out on the digital computer. The finite-difference method is the most well known of these techniques and is given a fairly comprehensive treatment in Chap. 8.

During the 1960s there was widespread development of the finite-element method for obtaining numerical solutions to problems. These developments have been primarily in the field of solid-body mechanics but have spilled over into heat transfer because of the thermal-stress problem. The finite-element method is presented in Chap. 9. The examples that are discussed are identical to those in Chap. 8 so that comparisons with the finite difference method can be made.

Several appendixes have been included to enable a student to take a conduction heat-transfer course on his own without benefit of an instructor. A selection of comprehensive problems is given in Appendix F to help provide experience in building mathematical models, using the mathematical techniques discussed in the text, and then drawing some engineering conclusions. Appendix G shows how these comprehensive problems could be used in a one-semester course. The comments in Appendix H should help the student select the most important points in each section of the text. The exercises at the end of each chapter can be worked as the student desires to provide a check on his understanding of the text material. Answers to many of these exercises are given in Appendix I.

ACKNOWLEDGMENTS FOR THE FIRST EDITION

The people most responsible for this book are the students for whom it is written. Their questions, suggestions, and criticisms have served as an invaluable stimulus in the development of the book.

A special debt of gratitude is due Prof. E. F. Obert of the University of Wisconsin for inspiring the original set of notes for his AIM program for off-campus students and for his continuing advice during the development of the notes into a textbook.

Professors W. M. Kays of Stanford University and H. C. Perkins of the University of Arizona have each used the notes in their conduction courses and have given many helpful suggestions. Lectures by Prof. W. Weaver of Stanford University on the finite-element method in structural analysis were useful in preparing Chap. 9.

The comments and criticisms of the finite-difference and the finite-element chapters given by C. B. Moyer and F. C. Weiler of the Aerotherm Corporation, P. S. Andersen of the Danish Atomic Energy Commission Research Establishment Risö (currently a Ph.D. candidate at Stanford), and B. F. Blackwell of the Sandia Corporation (currently a Ph.D. candidate at Stanford), based on their practical experience with computer techniques, have greatly improved this portion of the book.

This book could never have been produced without the assistance of the ladies who did the typing. Mary Huber has borne the brunt of all the typing, cutting, pasting, and retyping that occurred during the years of developing the final manuscript. Her patience, attention to detail, and typing skill have been a valued contribution. Jan Elliott, Bobbe Fowlie, Ditter Peschcke-Koedt, and Betsy Emory all contributed expertly in the final stages of preparation of the manuscript.

Finally, this book could never have been written if it were not for the support of my wife. Her encouragement and sacrifice have been an invaluable contribution.

GLEN E. MYERS

PREFACE TO THE SECOND EDITION

It has been 27 years since the first edition of this book was printed. A major change during this period has been the increase in computing power available to students and to engineers. The importance of numerical methods relative to analytical methods has increased. As a result, the one-semester course in heat conduction that I teach has evolved until it is about one-half analytical (primarily Bessel equations/functions, separation of variables and superposition) and one-half numerical (finite differences and finite elements). The class schedule in Appendix G shows the time I spend and the sections I cover. Complex combination and Laplace transforms are barely mentioned in my course to free up more time for numerical methods. Matrix notation and linear algebra are additional skills that are developed in studying numerical methods in this edition.

Although the major modifications of the second edition are in the finite-difference and finite-element chapters, there is a significant change in the emphasis in the analytical chapters. In the first edition, most of the mathematical solutions used normalized variables and parameters. Students were encouraged to first derive the governing equation (and its boundary conditions) in dimensional form and then normalize the problem before obtaining the mathematical solution. Unfortunately, students often thought that they *must* first normalize a problem before solving it (and sometimes normalization was a struggle). During the past 27 years I have come to believe that it is better to first solve the problem[1] and then normalize the solution for presentation. Dimensions, units and familiar parameter groups (or lack thereof) can often be used to help uncover math errors. It is easier to make wise normalization choices when you already have the solution. Therefore, in the second edition, most of the solutions are carried out in dimensional form. The benefits of normalization without solving the differential equation are still discussed in Chapter 7.

The treatment of separation of variables in Chapter 3 has been revised. Since most real-life heat-conduction problems are not homogeneous, Chapter 3 now approaches *all* problems as if they were nonhomogeneous rather than first discussing the special case of homogeneous problems. Section 3•1 first discusses one-dimensional, transient problems to illustrate the solution method[2] when the nonhomogeneous terms are independent of time. When the nonhomogeneous terms are time dependent another method (variation of parameters) is discussed. The discussion of two-dimensional problems in Section 3•2 is more than was covered in the first edition to set the stage for the discussion of numerical methods. Solutions for a steady-state problem and for a transient problem are found. These two analytical solutions are the test cases for the finite-difference and finite-element solutions in chapters 8 and 9. Section 3•3 treats four cylindrical-coordinate problems, one problem in spherical coordinates and then discusses the reduction of two- and three-dimensional transients to one-dimensional transients.

Most students now taking a graduate course in conduction have already had a reasonable exposure to one-dimensional, finite-difference solutions in their previous heat-transfer course(s). Thus the material on numerical methods in the second edition of this book primarily concentrates on two-dimensional problems. As in the first edition, the finite-difference method is discussed first (Chapter 8) because most students already have some background that makes it easier to introduce matrix notation, derive the governing finite-difference equations, review solution techniques (Gauss elimination, Cholesky decomposition, Euler and Crank-Nicolson) and understand numerically induced oscillations. It is also helpful to show some of the difficulties that arise in using finite differences for two-dimensional problems to motivate the need to study finite elements.

Chapter 8 now contains a major new section on the analysis of transient solutions. In this section the discrete finite-difference system of ordinary differential equations for transient problems is solved exactly (*i.e.*, without discretizing the time variable). The solution technique and notation used in Chapter 8 to obtain the exact solution of the transient finite-difference equations is similar to the method used in Chapter 3. Although the exact solution of the finite-difference system of differential equations is not yet practical for large conduction problems, it is instructive to compare it to solutions obtained by separation of variables in Chapter 3. The “exact” finite-difference solution is also compared to the Euler and Crank-Nicolson solutions to gain an understanding of the critical-time-step problem.

Chapter 9, on finite elements, has been completely rewritten. In the first edition, finite-element theory was based on variational calculus (which was new to most students). In this second edition, Galerkin’s method is used to develop finite elements because it is easier for students to understand. The governing finite-element system of ordinary differential equations uses notation introduced in Chapter 8 for finite differences. Chapter 9 is written

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1 This assumes that a solution is obtainable.

2 The solution method was called “partial solutions” in the first edition.

for easy comparison to the more familiar finite-difference material in Chapter 8.

Many of the exercises at the end of Chapter 9 now ask the student to use the computer program *FEHT*[1] to construct finite-element models and make calculations. *FEHT* has both Macintosh and Windows versions.

This second edition has been developed primarily over the past eight summers on a Macintosh computer. The large page size (to reduce the number of pages and to provide room for the large matrix equations in chapters 8 and 9) and the page format (wide margins for figures and student notes) were chosen early so the revised class notes produced every other year would closely resemble the final version of the book. Students have been able to study from the latest version of the developing text and could offer comments to improve the book. An index was included with the last set of revised class notes to obtain student feedback. Printer's plates were produced directly from the author's PDF files. Hopefully this process of development and production will enhance the accuracy of the final version of the book.

COMPUTER COMMUNICATION WITH THE AUTHOR

If you have questions, concerns or suggestions about this book, you can contact the author via e-mail at: *myers@engr.wisc.edu*.

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I am indebted to Professors S. A. Klein and W. A. Beckman of the University of Wisconsin-Madison for writing program *FEHT* to solve heat-conduction problems using the finite-element techniques discussed in Chapter 9. It is now convenient to assign problems for students to work that involve irregular geometries requiring hundreds of nodes to model. Many interesting questions may be studied with *FEHT*.

The computations and original plots for most of the graphs in the book and the tables in the appendixes were done using *EES*[2], another program written by Professor S. A. Klein.

Professors G. H. Golub of Stanford University and B. Noble of the University of Wisconsin-Madison provided many helpful ideas about matrix theory, linear algebra and numerical methods.

L. L. Litzkow's creativity, computer knowledge, graphics capabilities and her willingness to provide assistance saved me many times when I had trouble getting the computer to do what I wanted.

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[1] Klein, S. A., W. A. Beckman and G. E. Myers: *FEHT – Finite Element Analysis*, F-Chart Software <http : //www.fchart.com/>.

[2] Klein, S. A. and F. L. Alvarado: *EES – Engineering Equation Solver,* F-Chart Software <http : //www.fchart.com/>.